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1972 J. Phys. A: Gen. Phys. 5 L25

(http://iopscience.iop.org/0022-3689/5/2/014)

J. Phys. A: Gen. Phys., Vol. 5, February 1972. Printed in Great Britain

Letters to the Editor

The external field of a nonstatic isolated system in Bonnor's unified field theory

D N PANT

Department of Mathematics, Banaras Hindu University, Varanasi-5, India

MS received 18 October 1971

Abstract. The field equations of Bonnor's unified field theory have been considered to obtain the external field of a general nonstatic spherically symmetric isolated system containing electric and magnetic charges associated with matter. It has been found that there exists only one nontrivial solution and it represents a static isolated magnetic monopole.

As is well known the field equations of Einstein's unified field theory (1953) do not lead to the Coulomb force between charged particles and further they are not in favour of the existence of a nonstatic spherically symmetric isolated system containing charge (Rao 1955). So far as the first defect is concerned it has been remedied by Bonnor (1954) by proposing a generalized set of field equations. In Bonnor's unified field theory no attempt has yet been made to obtain an exact spherically symmetric solution. In the present investigation the possibility of the existence of a nonstatic spherically symmetric isolated system containing electric and magnetic charges has been explored in Bonnor's theory.

The total field of a nonstatic spherically symmetric distribution is given by

$$\begin{aligned} g_{11} &= -\alpha(r,t) \qquad g_{22} = g_{33}\operatorname{cosec}^2\theta = -\beta = -r^2 \\ g_{44} &= \gamma(r,t) \qquad g_{\underset{\vee}{14}} = \omega(r,t) \qquad g_{\underset{\vee}{23}} = f(r,t)\sin\theta. \end{aligned} \tag{1}$$

The charge-and-current vector density j^s is defined by (Einstein 1953)

$$j^s = \tfrac{1}{6}\eta^{ikls}(g_{\underset{\vee}{ik},l} + g_{\underset{\vee}{kl},i} + g_{\underset{\vee}{li},k}).$$

Since the external field is free from charge and current we have

$$j^1 \equiv \dot{f}\sin\theta = 0 \qquad j^2 = j^3 \equiv 0 \qquad j^4 \equiv f'\sin\theta = 0 \tag{2}$$

where a dot and a prime denote partial differentiation with respect to t and r respectively.

The field equations of Bonnor's theory are:

$$g_{ij,k}-g_{sj}\Gamma_{ik}{}^{s}-g_{is}\Gamma_{kj}{}^{s} = 0 \tag{3a}$$

$$\Gamma_{\underset{\vee}{is}}{}^{s} = 0 \tag{3b}$$

$$R_{\underline{ij}}+p^2 U_{\underline{ij}} = 0 \tag{3c}$$

$$R_{[\underset{\vee}{ij},k]}+p^2 U_{[\underset{\vee}{ij},k]} = 0 \tag{3d}$$

where p is a constant and

$$U_{ij} = g_{\underset{\vee}{ji}}-g^{\underset{\vee}{mn}}g_{im}g_{nj}+\tfrac{1}{2}g^{\underset{\vee}{mn}}g_{nm}g_{ij}. \tag{4}$$

The set of equations (3*b*) yield

$$-\frac{\omega}{\alpha}(\psi'-\phi') = 0 \qquad -\frac{\omega}{\gamma}(\dot{\psi}-\dot{\phi}) = 0 \tag{5}$$

where

$$\psi = \ln\left(1-\frac{\alpha\gamma}{\omega^2}\right) \qquad \phi = \ln(f^2+\beta^2). \tag{6}$$

The surviving equations in (3*c*,*d*) are

$$R_{11}+p^2U_{11} = 0 \tag{7a}$$

$$R_{22}+p^2U_{22} = \operatorname{cosec}^2\theta(R_{33}+p^2U_{33}) = 0 \tag{7b}$$

$$R_{44}+p^2U_{44} = 0 \tag{7c}$$

$$R_{\underline{14}} = 0 \tag{7d}$$

$$R_{\underset{\vee}{23}} = c\sin\theta \tag{7e}$$

where c is independent of r. Using (2), (5) and (6) we find that (7*d*) reduces to

$$\frac{\dot{\alpha}}{2\alpha}\phi'\left(1-\frac{\omega^2}{\alpha\gamma}\right) = 0 \tag{8}$$

which suggests that

$$\dot{\alpha} = 0. \tag{9}$$

Using (2), (4), (5), (6) and (9) we find that the other equations in (7) take the following form:

$$\frac{1}{2}\phi''+\left(\frac{\omega^2\psi'}{2\alpha\gamma}+\frac{\gamma'}{2\gamma}\right)'+(\phi')^2\frac{f^2+\beta^2}{8\beta^2}-\frac{1}{4\alpha^2\gamma^2}(\gamma\alpha'-\alpha\gamma'-\omega^2\psi')$$
$$\times(\alpha\gamma'+\omega^2\psi')+\frac{\phi'\alpha'}{4\alpha}+p^2\alpha\left(\frac{f^2}{f^2+\beta^2}-\frac{\omega^2}{\omega^2-\alpha\gamma}\right) = 0 \tag{10a}$$

$$-\left(\phi'\frac{f^2-\beta^2}{4\alpha\beta}\right)'-\frac{\phi'}{8\alpha\beta}(f^2-\beta^2)\frac{\partial}{\partial r}\{\ln(\omega^2-\alpha\gamma)\}$$
$$-1-(\phi')^2\frac{f^2}{4\alpha\beta}-p^2\beta\left(\frac{f^2}{f^2+\beta^2}-\frac{\omega^2}{\omega^2-\alpha\gamma}\right) = 0 \tag{10b}$$

$$-\left(\frac{\omega^2\psi'}{\alpha^2}+\frac{\gamma'}{2\alpha}\right)'-(\phi')^2(f^2+\beta^2)\frac{\omega^2}{8\alpha^2\beta^2}$$

$$-\frac{1}{4\alpha^3\gamma}(\alpha\gamma'+2\omega^2\psi')(\gamma\alpha'-\alpha\gamma'-\omega^2\psi'+\alpha\gamma\phi')-(\psi')^2\left(\frac{\omega}{2\alpha}\right)^2$$

$$-p^2\gamma\left(\frac{f^2}{f^2+\beta^2}-\frac{\omega^2}{\omega^2-\alpha\gamma}\right)=0 \qquad (10c)$$

$$-\left(\frac{f\phi'}{2\alpha}\right)'-\frac{f\phi'}{4\alpha^2\gamma}(\gamma\alpha'+\alpha\gamma'+\omega^2\psi')+f(\phi')^2\frac{f^2-\beta^2}{8\alpha\beta^2}$$

$$-p^2f\left(\frac{f^2+2\beta^2}{f^2+\beta^2}+\frac{\omega^2}{\omega^2-\alpha\gamma}\right)-c=0. \qquad (10d)$$

From (10*a*) and (10*c*) we get

$$\frac{1}{2}\phi'\left(1-\frac{\omega^2}{\alpha\gamma}\right)\frac{\partial}{\partial r}\ln\left\{4\frac{\omega^2-\alpha\gamma}{\beta(\phi')^2}\right\}=0. \qquad (11)$$

Computing (10*b*) and (10*d*) and simplifying the result with the help of (5), (6) and (11) one gets

$$\beta^4(c+2p^2f)+f\beta^2(4cf+7p^2f^2-p^2l^4)+4f^3\beta$$
$$-f^3(cf+p^2f^2-p^2l^4)=0 \qquad (12)$$

where l is a constant. In view of (1) the equation (12) will hold if $f=c=0$. This relation satisfies (10*d*) identically. Solving the remaining equations in (10) and using (1) we finally obtain the following solution:

$$\alpha=\left\{1-\frac{2m}{r}-p^2\left(\frac{l^4}{r^2}\right)\right\}^{-1} \qquad \beta=r^2$$
$$\gamma=\left(1+\frac{l^4}{r^4}\right)\left\{1-\frac{2m}{r}-p^2\left(\frac{l^4}{r^2}\right)\right\} \qquad \omega=\pm\frac{l^2}{r^2} \qquad f=0. \qquad (13)$$

This solution represents a massive magnetic monopole at rest. Further it is to be noted that this is the only possible exact spherically symmetric solution for an isolated system in Bonnor's theory. Thus we conclude that, like Einstein's theory, Bonnor's theory also does not permit a nonstatic isolated system containing charge to exist. Moreover, it has been proved earlier (Tiwari and Pant 1971) that the static spherically symmetric solution representing the external field of an isolated charged distribution does not exist in the theory. In view of the above, Bonnor's theory seems to have an unsatisfactory feature in common with Einstein's theory.

The author is grateful to Dr R Tiwari for his guidance in the investigation. His thanks are also due to Dr S R Roy for useful discussions.

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